

# SAT Cram Session: Math You Need to Know

## Problems We Work Together

### I. Content

#### Algebra

##### System of Equations: FAVORITE!

##### *Calculation*

##### Substitution and Elimination

1. For the system of equations shown below, what is the value of  $x$ ?

$$2x + 3y = 3$$

$$x - y = 4$$

2. For the system of equations shown below, what is the value of  $x$ ?

$$6x - y = 23$$

$$4y - 2x = 18$$

##### Elimination is necessary

3. What is the solution  $(x, y)$  to the system of equations shown below?

$$10x - 8y = 4$$

$$-5x + 3y = -9$$

One more, for practice...

4. What is the solution  $(x, y)$  to the system of equations shown below?

$$2x + y = 10$$

$$8x - 2y = 4$$

- A) (5, 0)
- B) (2, 6)
- C) (1, 2)
- D) (0, 0)

## Double Elimination

5. What is the solution  $(x, y)$  to the system of equations shown below?

$$5x + 3y = 20$$

$$11x + 5y = 28$$

Another:

6. What is the solution  $(x, y)$  to the system of equations shown below?

$$-4x - 3y = -13$$

$$7y + 9x = 32$$

Here's one I've seen occurring with greater frequency:

$$4x - 2y = 7$$

$$5x + y = 3$$

7. If  $(x, y)$  is the solution to the above system of equations, what is the value of  $9x - y$  ?

*Conceptual*

"No Solution"

$$-3y + ax = 36$$

$$2y - 10x = 14$$

8. If  $a$  is a constant, for what value of  $a$  will the system of equations have no solution?

Let's do another:

$$dx + 4y = 16$$

$$15x - 5y = 30$$

9. If  $d$  is a constant, for what value of  $d$  will the system of equations have no solution?

$$y = 3x + 4$$

10. The above equation is one of two equations in a linear system that has no solution. Which of the following equations could be the second equation in the system?

- A)  $y = \frac{1}{3}x + 4$
- B)  $y = \frac{1}{3}x - 3$
- C)  $y = 3x - 3$
- D)  $y = 3x + 4$

### “Infinite Solutions”

$$3y - 6x = 12$$

$$ty + zx = 48$$

11. For the system of equations above,  $t$  and  $z$  are constants. If the system of equations has infinite solutions, what is the value of  $t/z$  ?

$$2x + 14y = 8$$

$$10x + 70y = 10c$$

12. For what value of  $c$  would the following system of equations have an infinite number of solutions?

### Some other varieties of “no solution” and “infinite solutions”

$$y = 4x + 7$$

$$y = kx + 2$$

13. For the system of equations shown above,  $k$  is a constant. If the system has no solutions, then which of the following could be the value of  $k$ ?

- I. 2
  - II. 4
  - III. 5
- A) I only
  - B) II only
  - C) III only
  - D) Neither I, II, nor III

## Quadratics in General

14. What are the solutions to the equation  $x^2 - 10x + 21 = 0$  ?

### **MEMORIZE: the 3 keys to quadratics**

- I. Factor and find the solutions**
  - a. Synonyms for solutions: zeroes, x-intercepts**
- II. Find the vertex**
  - a. Use  $-\frac{b}{2a}$  or halfway between zeroes**
  - b. Vertex is also called “minimum” or “maximum”**
- III. Find the y-intercept**

Back to the question....

14a. What are the solutions to the equation  $x^2 - 10x + 21 = 0$  ?

b. What is the vertex?

c. What is the y-intercept?

Let's do another one together:

15a. What are the solutions to  $x^2 + 12x + 20 = 0$  ?

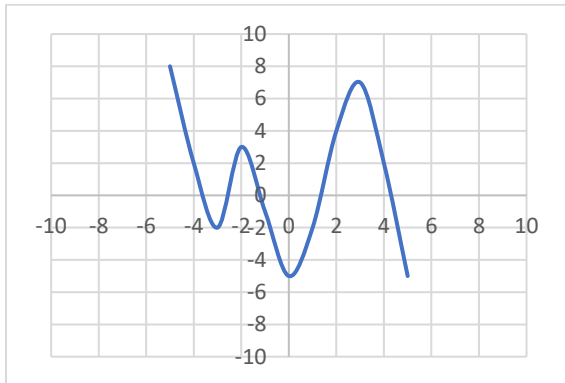
b. What is the vertex?

c. What is the y-intercept?

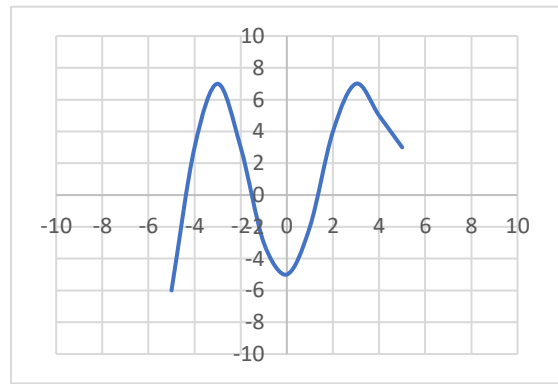
HERE ARE A FEW WAYS THE SAT LIKES TO TEST THESE CONCEPTS:

16. If a function has four distinct zeroes, which of the following could be the function of the graph?

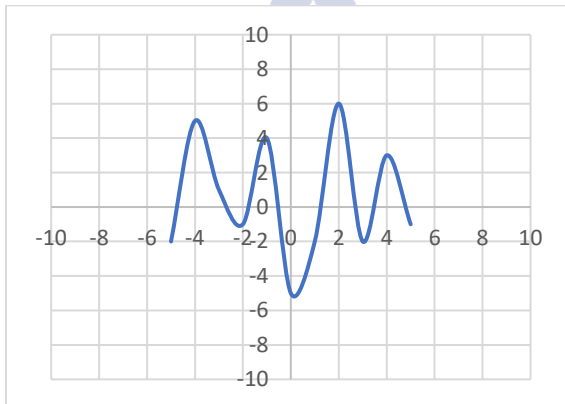
A.



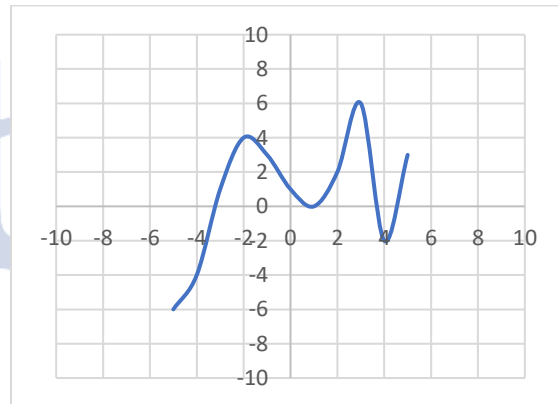
B.

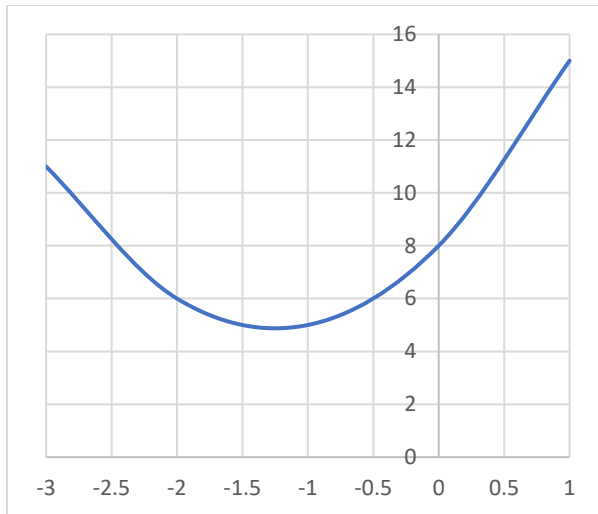


C.



D.





17. For the graph of  $y = 2x^2 + 5x + 8$ , which of the following is the y-intercept of the graph?

- A) -3
- B) 0
- C) 8
- D) 15

18. For the function  $f(x) = (x + 5)(x - 3)$ , which of the following gives the x-coordinate of the vertex of the graph of  $f(x)$  ?

- A) -2
- B) -1
- C) 0
- D) 1

19. For  $f(x) = x^2 + 14x + 33$ , which of the following is the minimum value of the function?

- A) (-7, -16)
- B) (-3, 33)
- C) (7, 180)
- D) (11, 462)

20. If the function  $f(x) = x^2 - 6x + 8$  has two x-intercepts, what is the sum of the x-coordinates of the x-intercepts?

- A) -6
- B) -4
- C) 4
- D) 6

## Quadratic Formula

### MEMORIZE:

For the standard form of a quadratic equation,  $ax^2 + bx + c = 0$ ,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

21. What are the solutions to  $3x^2 - 24x + 12 = 0$  ?

- A)  $4 \pm 2\sqrt{3}$
- B)  $8 \pm 4\sqrt{3}$
- C)  $-3 \pm 4\sqrt{3}$
- D)  $-4 \pm 2\sqrt{3}$

22. What are the solutions to  $\frac{1}{4}x^2 - \frac{1}{2}x - \frac{1}{8} = 0$  ?

- A)  $-1 \pm 2\sqrt{3}$
- B)  $1 \pm \frac{\sqrt{6}}{2}$
- C)  $-4 \pm \frac{\sqrt{8}}{2}$
- D)  $4 \pm 2\sqrt{6}$

$$5x^2 - 60x + 15 = 0$$

23. What is the sum of the solutions for the equation above?

## Exponent Rules

### MEMORIZE:

Most important:  $x^0 = 1$

$$x^1 = x$$

$$x^a x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Often utilized on the hardest problems:

$$x^{-a} = \frac{1}{x^a}$$

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

24. Which of the following is equal to  $d^{\frac{4}{5}}$ , for all values of  $d$ ?

- A)  $\sqrt[4]{d^5}$
- B)  $\sqrt{d^5}$
- C)  $\sqrt{d^4}$
- D)  $\sqrt[5]{d^4}$

$$(\sqrt[3]{x^4})^z, \text{ where } x \geq 0$$

25. In the expression above,  $z$  is a constant. The expression is equivalent to  $x^{12}$ , where  $x \geq 0$ . What is the value of  $z$ ?

26. Which of the following is equivalent to  $\sqrt{36c^{36}}$ ?

- A)  $6\sqrt{c^6}$
- B)  $6c^{18}$
- C)  $6c^6$
- D)  $\sqrt{18c^{18}}$

27. Which of the following is equivalent to  $25^{\frac{2}{3}}$ ?

- A)  $25^3\sqrt{25^2}$
- B)  $25^2\sqrt{25^2}$
- C)  $5^3\sqrt{5}$
- D)  $5^2\sqrt{5^3}$



$$\frac{a^{\frac{2}{3}}b^{-4}}{a^{-3}b^{\frac{1}{5}}}$$

28. The expression above is equivalent to which of the following?

- A)  $\frac{a^3\sqrt[3]{a^2}}{b^4\sqrt[5]{b}}$
- B)  $\frac{\sqrt[3]{a^5}}{\sqrt[5]{b^4}}$
- C)  $\frac{a}{b}$
- D)  $\frac{a^3}{b^5}$

## Exponential Growth/Decay

$$D(t) = 100 (1.08)^t$$

29. The function above models the number of dandelions,  $D(t)$ , in a field  $t$  days after an initial count of the dandelions in the field was made on March 15<sup>th</sup>. Which of the following does 1.08 in the function represent?

- A) There were 1.08 dandelions in the field on March 15<sup>th</sup>.
- B) The dandelion population of the field grew by 1.08 each day.
- C) The dandelion population grew by a factor of 1.08 each day.
- D) It takes 1.08 days for the field's dandelion population to double.

30. A certain town had a population of 20,000 residents in 2005, and the town's population has declined by 6% annually since then. Which of the following functions models the population of the town,  $P(t)$ , after  $t$  years?

- A)  $20,000 (.06)^t$
- B)  $20,000 (.94)^t$
- C)  $20,000 (1.06)^t$
- D)  $20,000 (1.94)^t$

31. A researcher estimates that the population of a certain strain of bacteria will grow by 70% every 20 minutes. If there is a starting population of 100,000 bacteria, which of the following expressions best represents the population of the bacteria colony  $m$  minutes from now?

- A)  $B(t) = 100,000 (1.70)^{20m}$
- B)  $B(t) = 100,000 (1.70)^{\frac{m}{20}}$
- C)  $B(t) = 100,000 (.70)^{20m}$
- D)  $B(t) = 100,000 (.70)^{\frac{m}{20}}$

## Imaginary and Complex Numbers

### MEMORIZE:

$i^2 = -1$  (The test will tell you that  $i = \sqrt{-1}$ , but the test will not tell you that  $i^2 = -1$ )

32.  $(7 + 12i) + (2 - 3i) =$

- A)  $14 - 36i$
- B)  $5 + 9i$
- C)  $9 - 36i$
- D)  $9 + 9i$

33. What is the sum of  $10 + 11i$  and  $8 + 4i$ ?

- A)  $2 + 7i$
- B)  $2 + 15i$
- C)  $18 + 7i$
- D)  $18 + 15i$

34.  $(17 + 11i) - (6i^2 - 5i) =$

- A)  $23 + 6i$
- B)  $23 + 16i$
- C)  $17 + 11i$
- D)  $11 + 16i$

35.  $(16 + 7i) - (-2i^2 - 3i) =$

- A)  $14 + 10i$
- B)  $14 + 4i$
- C)  $18 + 10i$
- D)  $18 + 4i$

# Geometry

## MEMORIZE:

Remember: the SAT gives you a full page of formulas at the start of each math test. Do not forget that it gives you the Pythagorean Theorem as well as the side lengths for a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle as well as a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle.

2 right triangles the test loves: 3-4-5 and 5-12-13. If there's a right triangle in a problem with a side length/side lengths given, instantly check to see if it's one of these two triangles.

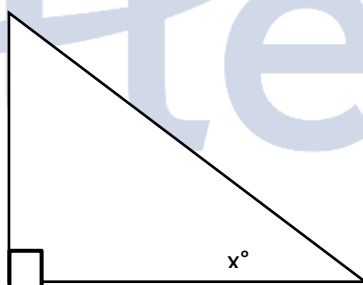
Important terminology: similar vs. congruent triangles

If two triangles are similar, that means they have the same angles measures, but not necessarily the same side lengths

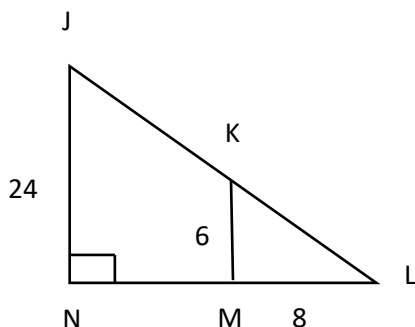
If two triangles are congruent, then both their angle measures and side lengths are the same.

Trig functions: sine, cosine, and tangent

$$\begin{array}{ccc} \text{SOH} & \text{CAH} & \text{TOA} \\ \sin x^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} & \cos x^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} & \tan x^\circ = \frac{\text{Opposite}}{\text{Adjacent}} \end{array}$$



36. In the figure below,  $\overline{KM}$  is parallel to  $\overline{JN}$ . What is the length of  $\overline{JL}$  ?

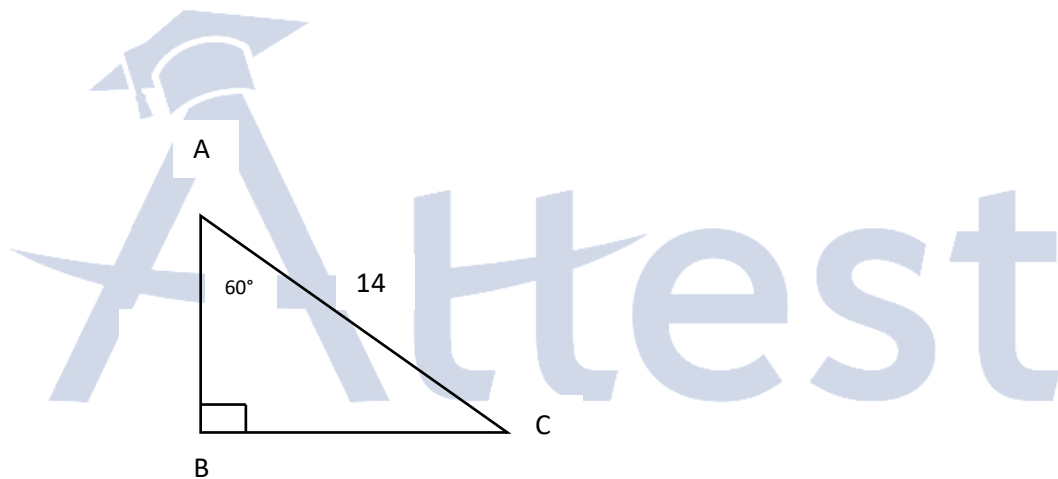


37. For right triangle ABC with right angle B,  $\cos A = \frac{3}{5}$ . What is the  $\sin C$ ?

38. For right triangle TUV, angle U is a right angle and side TU has a length of 18. Which of the following additional pieces of information can be used to determine the length of side UV?

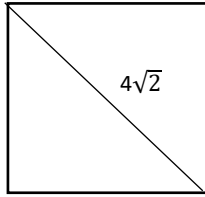
- I. The measure of angle T
- II. The length of side TV

- A) I only
- B) II only
- C) Both I and II
- D) Neither I nor II



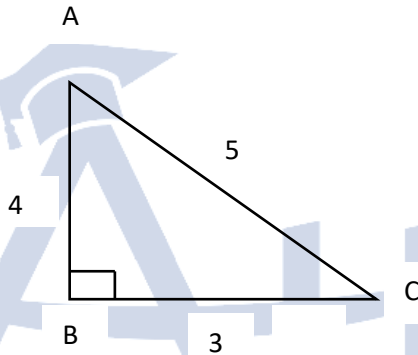
39. For triangle ABC above, what is the length of side BC?

- A) 7
- B)  $7\sqrt{2}$
- C)  $7\sqrt{3}$
- D)  $14\sqrt{3}$



40. If the square shown above has a diagonal with length  $4\sqrt{2}$ , what is the area of the square?

$\sin x^\circ = \cos (90 - x)^\circ$



41. If  $\sin x^\circ = \frac{3}{5}$ , then the  $\cos (90 - x)^\circ = \underline{\quad}$ ?

42. If  $\sin 30^\circ = \frac{1}{2}$ , then the  $\cos 60^\circ = \underline{\quad}$ ?

43. If  $\sin x^\circ = d$ , then which of the following must be true?

- A)  $\cos (90 - x)^\circ = d$
- B)  $d = \cos (x)^\circ$
- C)  $\cos (x)^\circ = \sin d$
- D)  $\cos d = \sin d$

## Radians

If you've never heard of radians, then that's okay: it's easy to learn.

There are two ways we can measure angles: degrees and radians. We use degrees in everyday life, and that unit of measurement is based on the idea of there being  $360^\circ$  in a circle (and, by logical extension,  $180^\circ$  in a semicircle).

For radians, you just need to know that it is based on  $\pi$ , and that  $180^\circ = \pi$  radians.

Given that, if  $180^\circ = \pi$  radians, then...

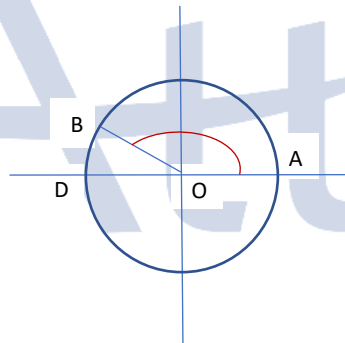
$$360^\circ = 2\pi \text{ radians}$$

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

...and so on.

Here are some problems you may see on the test:

44. If  $540^\circ = d\pi$  radians, then  $d = \underline{\hspace{1cm}}$  ?



45. In the graph above, point O is at the origin, points A and D lie on the x-axis, and the measure of angle BOD is  $30^\circ$ , the what is the radian measure of angle AOB?

- A)  $\frac{\pi}{6}$
- B)  $\frac{2\pi}{3}$
- C)  $\frac{5\pi}{6}$
- D)  $5\pi$

## Equation of a Circle

### MEMORIZE:

A) For a circle with center  $(h, k)$ , the equation for graphing that circle is:

$$(x - h)^2 + (y - k)^2 = \text{Radius}^2$$

46. For a circle with equation  $(x - 3)^2 + (y + 6)^2 = 100$ , what is the coordinate point for the center of the circle as well as the radius?

- A) Center  $(-3, 6)$ , Radius = 100
- B) Center  $(-3, 6)$ , Radius = 10
- C) Center  $(3, -6)$ , Radius = 100
- D) Center  $(3, -6)$ , Radius = 10

47. For a circle with equation  $(x + 5)^2 + (y + 11)^2 = 81$ , what is the coordinate point for the center of the circle as well as the radius?

- A) Center  $(5, 11)$ , Radius = 81
- B) Center  $(-5, -11)$ , Radius = 81
- C) Center  $(-5, -11)$ , Radius = 9
- D) Center  $(5, 11)$ , Radius = 9

48. If a circle has a center located at  $(8, -2)$  and a radius of 8, which of the following is the equation for that circle?

- A)  $(x - 8)^2 + (y - 2)^2 = 4$
- B)  $(x + 8)^2 + (y + 2)^2 = 2$
- C)  $(x - 8)^2 + (y + 2)^2 = 64$
- D)  $(x + 8)^2 + (y - 2)^2 = 16$

49. If a circle has a center located at  $(-9, -12)$  and a radius of 5, which of the following is the equation for that circle?

- A)  $(x + 9)^2 + (y + 12)^2 = 5$
- B)  $(x + 9)^2 + (y + 12)^2 = 25$
- C)  $(x - 9)^2 + (y - 12)^2 = 5$
- D)  $(x - 9)^2 + (y - 12)^2 = 25$

$$x^2 + 12x + y^2 - 6y = 4$$

50. The graph of the equation shown above is a circle. What is the radius of the circle?

- A) 6
- B) 7
- C) 8
- D) 9

$$x^2 - 8x + y^2 - 2y = -1$$

51. The graph of the equation shown above is a circle. What is the radius of the circle?

- A) 4
- B) 5
- C) 6
- D) 7

## Probability and Statistics

### Standard Deviation

52. The following are the lengths of fish caught on two separate days, Saturday and Sunday.

Saturday:

21	23
18	25
22	20
20	19
21	21

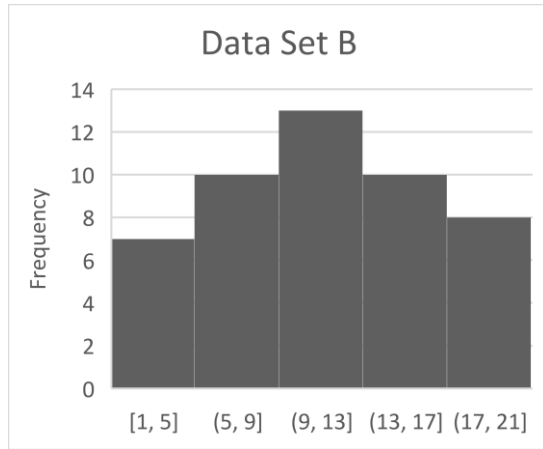
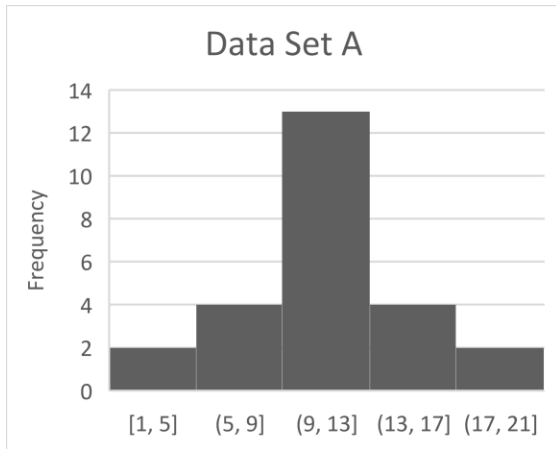
Sunday:

21	30
28	12
24	25
15	17
17	21

Based on the data, which of the following statements must be true?

- A) The standard deviation for Saturday is larger
- B) The standard deviation for Sunday is larger
- C) The standard deviations for Saturday and Sunday are the same
- D) The standard deviation cannot be determined for either date

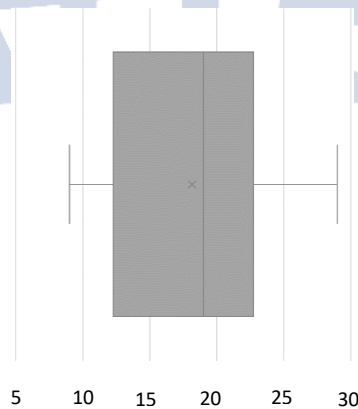




53. For data sets A and B shown above, which of the following best describes the relationship between the standard deviations of the two data sets?

- A) The data sets have equivalent standard deviations
- B) The standard deviation of data set A is greater than the standard deviation of data set B
- C) The standard deviation of data set B is greater than the standard deviation of data set A
- D) The standard deviations for the data sets cannot be determined

### Box Plots

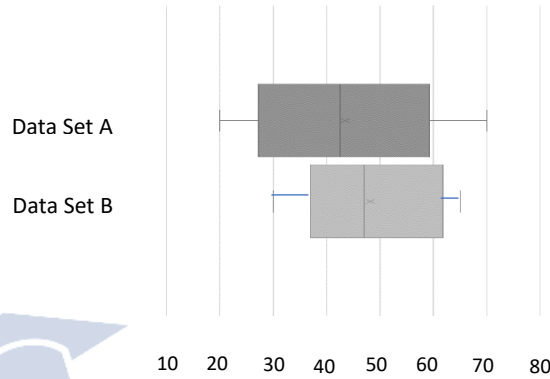


54. For the box plot shown above, which of the following is closest to the range for the data set?

- A) 8
- B) 10
- C) 20
- D) 25

55. For the box plot shown in the previous question, which of the following best approximates the median of the data set?

- A) 8
- B) 12
- C) 19
- D) 23



56. The box plots above represent two data sets. Which of the following statements is NOT true?

- A) Data set A has a greater range than Data set B
- B) Data set A has a smaller median than Data set B
- C) The minimum value of Data set A is smaller than the minimum value of Data set B
- D) The maximum value of Data set A is smaller than the maximum value of Data set B

## Statistical Analysis of Mean and Median

Number of Seats	Frequency
2	3
3	6
4	7
5	9
6	17
7	7
8	5

57. An airport parking lot currently has 54 cars on the lot. The frequency table shown above gives the number of seats in each car. If a car with 8 seats is added to the parking lot, how will this affect the mean and median of the overall data set?

- A) The mean will increase and the median will decrease.
- B) The mean will decrease and the median will increase.
- C) The mean will increase and the median will remain the same.
- D) The mean will remain the same and the median will increase.

## II. Test-taking Strategy

### Using Your Answers with Algebra

*Plug in the answers*

58. If  $\frac{17-x}{4} = 3$ , what is the value of  $x$  ?

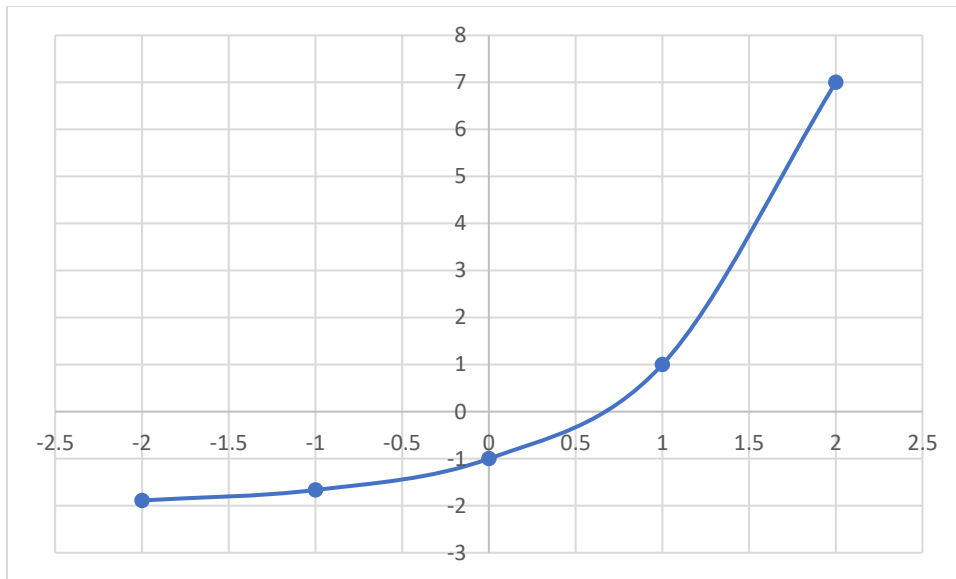
- A) 1
- B) 4
- C) 5
- D) 9

59. What are the solutions to  $x^2 - x - 20 = 0$  ?

- A) -1, -20
- B) 1, 20
- C) 4, -5
- D) -4, 5

## Solving Problems Involving Graphs and Data Tables

Graphs:

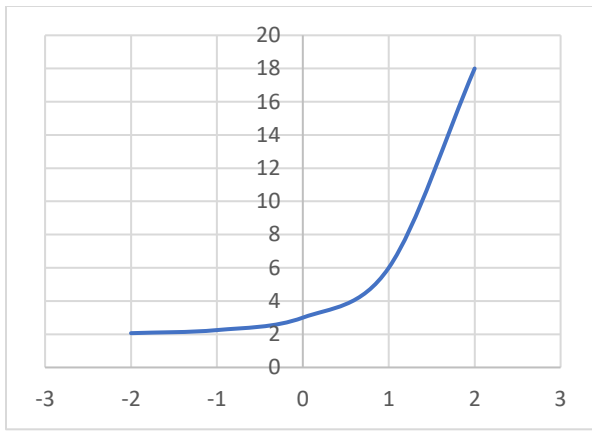


60. Which of the following equations represents the graph shown above?

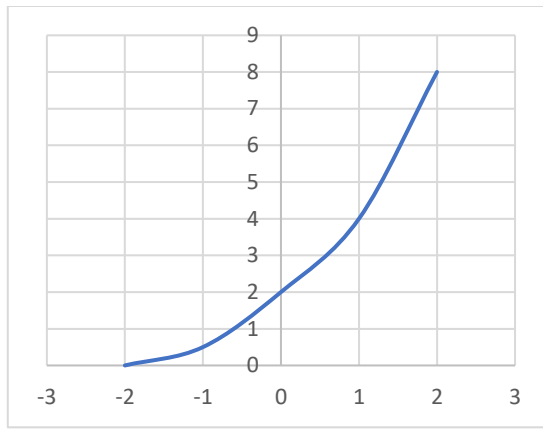
- A)  $y = 4^x - 2$
- B)  $y = 3^x - 2$
- C)  $y = 3^x - 1$
- D)  $y = 4^x - 1$

61. What is the graph for the equation  $y = 4^x + 2$  ?

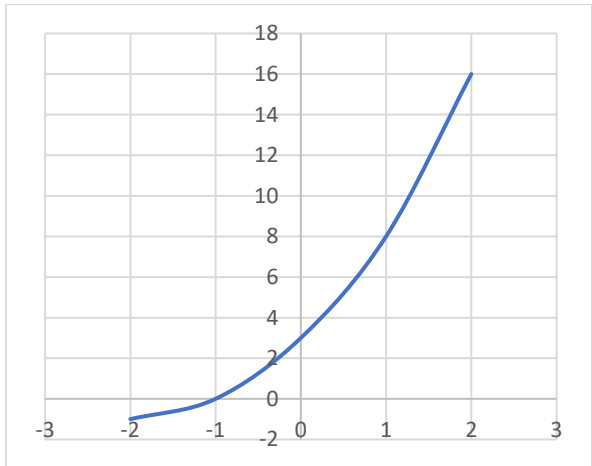
A.



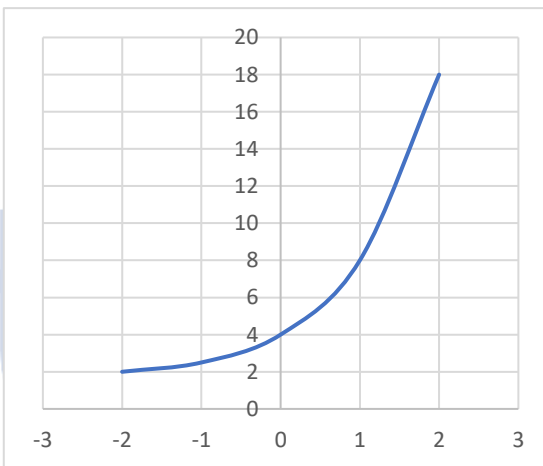
B.



C.



D.



Tables:

m	1	2	3	4	5
f(m)	-4	0	4	8	12

62. For the data table above, which of the following defines  $f(m)$  for the values listed?

- A)  $m - 5$
- B)  $2m - 6$
- C)  $4m - 8$
- D)  $6m - 10$

Rodent Infestation Time (days)	Number of Rodents
0	6
2	7
4	10
6	15

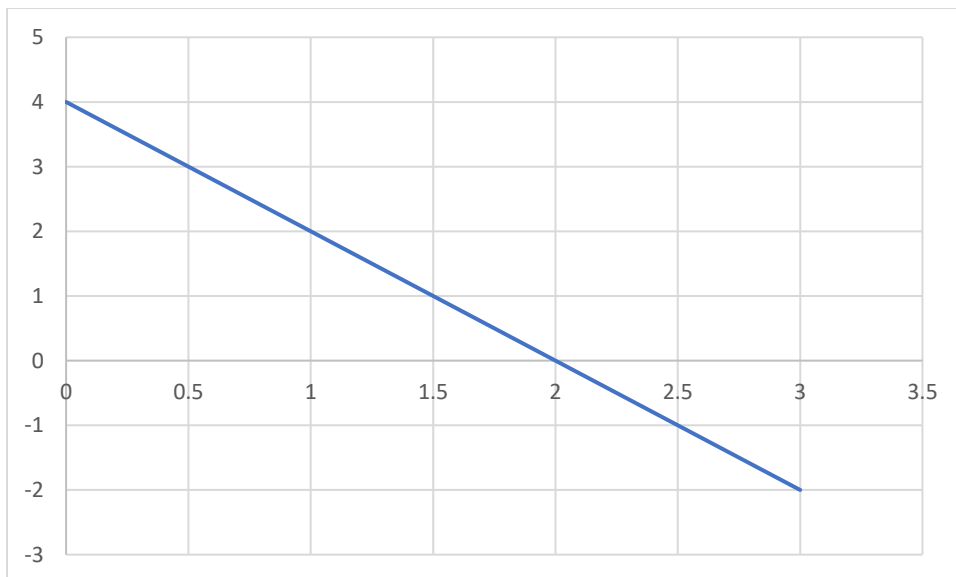
63. The table shown above gives some values for the number of rodents present in a house that house a rodent infestation  $x$  days since the infestation began. Which of the following equations correctly models this relationship?

- A)  $\frac{1}{4}x^2 - 6$
- B)  $\frac{1}{4}x^2 + 6$
- C)  $2x + 6$
- D)  $2x - 6$

### Using Answers to Get the Answer

64.  $10x^2 + 40x - 65 - 18x^2 - 6x + 37$  is equivalent to:

- A)  $-28x^2 + 34x + 37$
- B)  $-8x^2 + 34x - 28$
- C)  $8x^2 - 46x + 37$
- D)  $28x^2 - 46x - 28$



65. Which of the following is the slope the line graphed above?

- A) -2
- B)  $-\frac{1}{2}$
- C)  $\frac{1}{2}$
- D) 2

66. The expression  $(4x^5)^3$  is equivalent to which of the following?

- A)  $12x^8$
- B)  $12x^{15}$
- C)  $64x^8$
- D)  $64x^{15}$

67. If  $x > -1$ , which of the following is equivalent to  $\frac{1}{\frac{1}{x+1} + \frac{1}{x+4}}$  ?

- A)  $2x + 5$
- B)  $x^2 + 5x + 4$
- C)  $\frac{2x+5}{x^2+5x+4}$
- D)  $\frac{x^2+5x+4}{2x+5}$

## Algebra and "Equivalent"

*Easier:*

68. Which of the following expressions is equivalent to  $5b^3 - 10b^2$  ?

- A)  $-5b^2$
- B)  $-10b^2(2b + 1)$
- C)  $5b^2(b - 2)$
- D)  $b^2(-5 + b)$

69. If  $a > 0$ , which of the following is equivalent to  $\frac{2}{a} + \frac{4}{3a}$  ?

- A)  $\frac{10}{3a}$
- B)  $\frac{6}{3a}$
- C)  $\frac{6}{4a}$
- D)  $\frac{8}{3a^2}$

Difficult:

$$\frac{1}{6x+7} + 4$$

70. Which of the following is equivalent to above expression for  $x > 0$  ?

- A)  $\frac{6x+8}{6x+7}$
- B)  $\frac{24x+28}{6x+7}$
- C)  $\frac{24x+29}{6x+7}$
- D)  $\frac{5}{6x+7}$

71. If  $x > 2$ , which of the following is equivalent to  $\frac{3x^2-17x}{3x-5}$  ?

- A)  $x$
- B)  $x - 2 - \frac{17}{3x-5}$
- C)  $x - 3 - \frac{5}{3x-5}$
- D)  $x - 4 - \frac{20}{3x-5}$

Plugging-in for two variables:

72. Which of the following is equivalent to the expression  $(x + \frac{y}{4})^2$  ?

- A)  $x^2 + \frac{y^2}{16}$
- B)  $x^2 + \frac{xy}{2} + \frac{y^2}{16}$
- C)  $x^2 + \frac{xy}{4} + \frac{y^2}{4}$
- D)  $x^2 + \frac{xy}{4} + \frac{y^2}{16}$



## Does It Make Sense?

73. Jerome is a MRI technician at a local hospital. Each week, he has a certain number of patients in need of MRI imaging. The number of patients he has left at the end of each day can be estimated with the equation  $P(d) = 36 - 8d$ , where  $P$  is the number of patients left and  $d$  is the number of days he has worked that week. What is the meaning of the value 36 in this equation?

- A) It will take Jerome 36 days to see each patient.
- B) Jerome starts each week with 36 patients to see.
- C) Jerome sees 36 patients per hour.
- D) Jerome sees 36 patients per day.

